A New Hybrid FDTD-BIE Approach to Model Electromagnetic Scattering Problems

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***Abstract*—A new hybrid finite-difference time-domain boundary integral equation (FDTD-BIE) method is proposed. The geometry under consideration is decomposed into several subregions. For some of the bounded subregions the FDTD technique is used to construct an interaction matrix, relating virtual electric and magnetic currents at the boundary of the subregion, for several frequencies at the same time. The remaining bounded and unbounded subregions are described by a boundary integral equation technique. An example is given to validate the new technique.**

***Index Terms*— FDTD method, integral equations, numerical techniques.**

I. Introduction

WELL-KNOWN methods to solve electromagnetic scattering problems are the finite-element (FE), the boundary integral equation (BIE), and the finite-difference time-domain (FDTD) technique. FDTD differs substantially from the two prior ones. First, it is a time-domain technique (FE and BIE are more often used in the frequency domain). Second, by working iteratively instead of solving a matrix equation, the FDTD has low memory requirements, but each new excitation requires a new simulation. FE and BIE construct an interaction matrix belonging to a specific geometry, enabling to model different excitations using the same interaction matrix. A subregion's interaction matrix can also be reused for identical subregions in the simulation domain.

In this letter we combine the advantages of both FDTD and BIE techniques by choosing the BIE as global modeling technique and by using FDTD to construct the interaction matrix of appropriate subdomains at multiple frequencies. This new method, based on domain decomposition, also allows a more flexible gridding scheme than the global uniform mesh most often used by the FDTD technique.

Although a large number of hybrid methods has already been proposed, the only paper known to us concerning combining FDTD with BIE in the frequency domain was by Taflove [1]. Taflove's formalism, however, consists of a Born approximation restricted to aperture penetration with low coupling between internal and external regions. In our letter, multiple heavily coupled open regions are treated rigorously.

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II. Hybrid FDTD-BIE Formulation

Consider a sample three-dimensional (3-D) configuration of scatterers, excited by a plane wave with wave vector $\mathbf{k}_{\rm inc}$ [Fig. 1(a)]. The problem is partitioned into two subregions V_a and V_b by introducing a virtual interface S [Fig. 1(b]. Using field equivalence [2] we can describe the subregions separately by introducing virtual sources $\mathbf{J}_{\rm virt} = \hat{\mathbf{n}} \times \mathbf{H}$ and $\mathbf{M}_{\rm virt} = \mathbf{E} \times \hat{\mathbf{n}}$ on S. These virtual sources are the unknowns in our problem. Now we can use the most suitable technique for each subregion. In the past BIE and FE have been used successfully; here we will combine FDTD with BIE.

To model a subregion we express the fields generated in V_a and V_b by the virtual sources. The unknown sources are then found by imposing continuity between the tangential components of these fields at S.

A. Boundary Integral Equation Technique

A BIE [Fig. 1(d)] is most suited for the open and homogeneous external region V_b, provided the Green's function incorporates the radiation condition. The electric and magnetic fields at the interface S due to the virtual sources are calculated by means of the Poggio and Miller [3] 3-D integral expressions. To calculate the fields, the boundary S is discretized into rectangles and the unknown virtual sources, or equivalently tangential fields, are represented using rooftop basis functions $\mathbf{w}_r$

$$\mathbf{E}(\mathbf{r}) = \sum_{r=1}^{N} E_r \mathbf{w}_r(\mathbf{r}) \mathbf{H}(\mathbf{r}) = \sum_{r=1}^{N} H_r \mathbf{w}_r(\mathbf{r}) \mathbf{r} \in S \qquad (1)$$

B. Using FDTD to Construct a Subregion's Interaction Matrix

Now consider the inhomogeneous internal subregion V_a, modeled using an FDTD technique. One FDTD simulation should yield the fields generated at surface S due to an excitation by $\mathbf{J}_{\rm virt}$ or $\mathbf{M}_{\rm virt}$ at this surface. Since $\mathbf{J}_{\rm virt}$ and $\mathbf{M}_{\rm virt}$ are expanded into rooftop functions we have to calculate these fields for each basis function, requiring one FDTD simulation per basis function. Fortunately, this simulation can be optimized by applying field equivalence in the following way: We know that by introducing the virtual current on S fields remains unchanged in V_a and become zero in V_b. This allows to change the geometry of V_b completely without affecting the fields in V_a. Classically, this change consists of embedding V_a within a perfectly electric conducting surface S_{PEC} closely to S (Schelkunoff's field equivalence [4]).

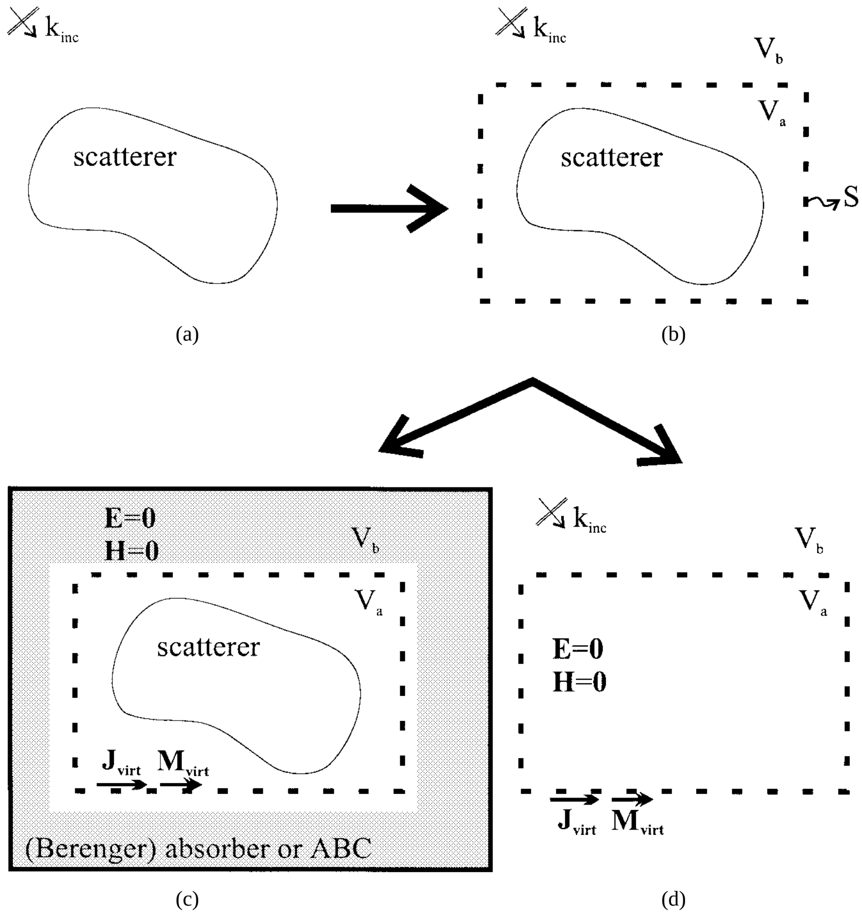

Fig. 1. Equivalence theorem.

However, this results in an inefficient FDTD simulation due to long-lasting transients. Instead of simply applying S_{PEC} we either use an absorbing boundary condition on S_{PEC} or use S_{PEC} coated with absorbing material [Fig. 1(c)], such as the PML, to terminate the FDTD grid in the region outside V_a.

Since FDTD is a time-domain technique, the fields at the boundary due to excitation of one basis function can be derived at multiple frequencies in one FDTD run. Each basis function is excited with a Gaussian time pulse. The resulting fields at the boundary S are then Fourier transformed to obtain the information at the desired frequencies [5]. After exciting all the rooftop basis functions, one obtains an FDTD interaction matrix for each required frequency. The matrices are stored on disk and for each excitation frequency, the correct FDTD interaction matrix is retrieved.

The size of one rooftop basis function extends over many Yee-cells in the FDTD algorithm. Such a basis function is then projected on the Yee mesh and implemented by electric and magnetic currents **J** and **M** in the FDTD scheme [6].

C. Coupling of FDTD and BIE Subregions

We impose continuity of the tangential fields at S in a weak sense through Galerkin weighting. This gives a matrix equation of the form $(\overline{\overline{Z}}_{\text{FDTD}} - \overline{\overline{Z}}_{\text{BIE}}) \cdot \mathbf{X} = \mathbf{B}$ with $\mathbf{X}$ containing the unknown coefficients E_r and H_r and $\mathbf{B}$ a source vector due to incoming fields.

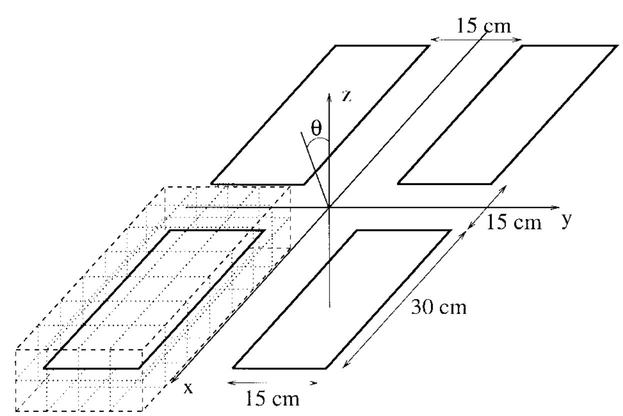

Fig. 2. Four-plate configuration.

Generalization of this formalism to multiple regions is straightforward. For each region an interaction matrix $\overline{\overline{Z}}$ is calculated with the prefered numerical technique. For identical subregions the $\overline{\overline{Z}}$ matrix is reused.

III. Example

To validate the proposed technique, we calculate the monostatic radar cross section of four infinitely thin perfectly conducting metallic plates (Fig. 2). Each plate is rectangular with dimensions 30 cm $\times$ 15 cm.

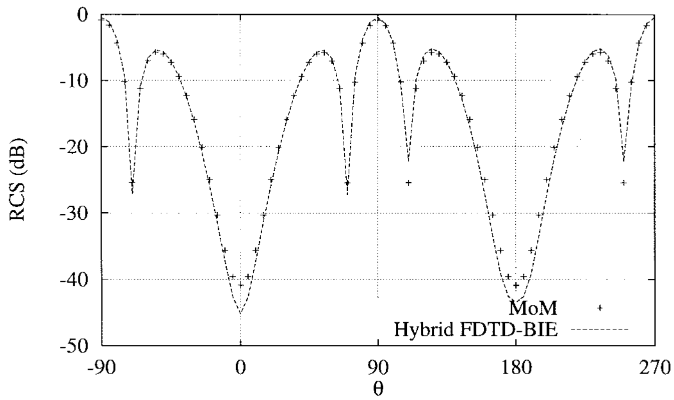

Fig. 3. Monostatic RCS in the xz plane for $f = 0.5$ GHz .

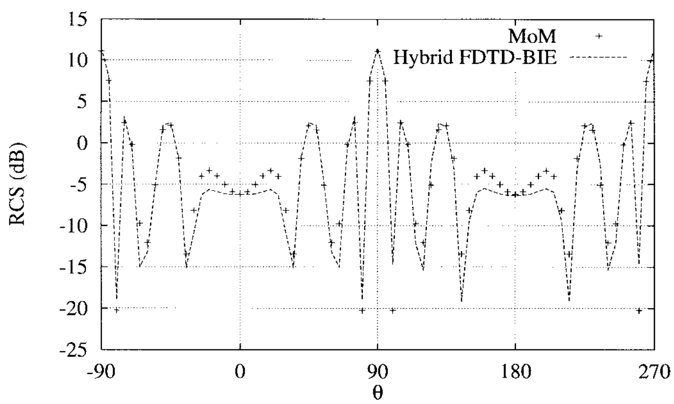

Fig. 4. Monostatic RCS in the xz plane for $f = 1$ GHz.

To model each metal plate separately with the FDTD method, a virtual interface is constructed around each one of them. Consider a subregion defined by the plate embedded within its virtual boundary. An FDTD interaction matrix is constructed following the technique of Section II-B. The subregion is covered by an FDTD grid with cell dimensions 1 cm $\times$ 1 cm $\times$ 1 cm and on its virtual interface a surface mesh is defined with resolution 7 $\times$ 4 $\times$ 2 (cell dimensions 5 cm $\times$ 5 cm). To minimize reflections at the FDTD grid termination, we add five layers of FDTD cells in each dimension and terminate the grid using a Mur second-order absorbing boundary condition. Exploiting symmetry, 64 FDTD simulations are needed to construct the interaction matrix for one plate within a frequency range from $f = 0.5$ GHz to $f = 1$ GHz. This matrix is reused for the three remaining plates since all plates are identical.

The monostatic RCS in the xz plane of the configuration (Fig. 2), at frequencies $f = 0.5$ GHz and $f = 1$ GHz, is shown in Figs. 3 and 4. The results obtained with the new hybrid method are in good agreement with those obtained from the Concept II MoM code [7].

IV. Conclusions

We presented a new hybrid FDTD-BIE technique for scattering problems. The FDTD method is used to construct an interaction matrix for certain subregions in the simulation domain. This requires a number of FDTD simulations equal to the number of rooftop basis functions defined on the boundary of the subregion under consideration. Once the interaction matrix for a subregion is known, however, it can be reused when that subregion is excited by different external excitations or when several identical subregions are present. Extra advantages compared to FDTD are that the radiation condition is rigorously taken into account by the BIE, no volume mesh is needed in BIE subregions, and gridding schemes are chosen for each of the FDTD subregions, separately.

Compared to the BIE, our technique can model highly complex and inhomogeneous bounded subregions efficiently with FDTD. When constructing an FDTD interaction matrix, each basisfunction is excited with a Gaussian pulse. After performing a Fourier transform, one obtains a row of the FDTD interaction matrix at a number of different frequencies in only one FDTD run. Finally, our technique requires less memory than hybrid FE-BIE techniques.

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